TOPIC: Learn about DEDUCTIVE and INDUCTIVE logic.

- Be able to recognize valid forms (modus ponens and modus tollens), and invalid forms (“affirming the consequent” and “denying the antecedent”).
- Know if arguments are valid and/or sound.
- Recognize the difference between deductive arguments and inductive arguments.

KEY TERMS/ GOALS:

- Definitions:
  - Deductive arguments
  - Validity
  - Valid forms: “Modus ponens” and “Modus tollens”
  - Invalid forms: “Affirming the consequent” and “denying the antecedent”
  - Counterexamples
  - Soundness
  - Inductive arguments

READING:

Vaughn ch. 2 (in E-reserves). Re-read this if you already read it. It is important.

Logic Practice Quiz (with answers) under “Handouts” folder, for practicing deductive logic.

CONTENT:

There are lots of kinds of arguments, but we will focus on two main types: Deductive and Inductive. (Some others are “Inference to the best explanation,” or “Reductio ad absurdum” which we will see later in the semester.) Deductive arguments make inferences from statements about general types or laws to statements about specific instances of the type or law. They are aimed at PROVING that the conclusion is true if the premises are true. Inductive arguments make inferences from more specific instances to more general types or laws. They only intend to show that the conclusion is PROBABLY true if the premises are true.

Deductive arguments have premises using general statements and conclusions about specific facts. Take the following argument: “This swan must be white because all swans are white,” outlined as follows:

- P: All swans are white.
- P: This is a swan.
- C: This swan is white.
I.e. If it is true that all swans are white, then I know that this particular swan is white.

Inductive arguments have specific premises and a general conclusion. I.e. People have observed (empirical evidence) 4 billion swans and they are all white. They conclude that ALL swans are white.

\[ P: \text{This swan is white.} \]
\[ P: \text{This swan is white. . . .} \]
\[ C: \text{ALL swans are white.} \]

**DEDUCTIVE LOGIC:**

*A deductive argument intends to offer logically conclusive support for its conclusion.* That is it intends to prove that something is true by following from the premises of the argument. The conclusion is intended to be true (not probably or mostly true, as an inductive argument might suppose).

If, for example, I say that “Roy is brave because he is a soldier and all soldiers are brave,” then I am intending to show that it is true, without fail, that Roy is brave since ALL soldiers are brave. Here is the argument outline:

\[ P_1. \text{All soldiers are brave. (All A’s are B’s)} \]
\[ P_2. \text{Roy is a soldier. (X is A. That is, since Roy is a soldier, he is a particular instance of A)} \]
\[ C. \text{Therefore, Roy is brave. (X must be a B)} \]

Deductive arguments have a special feature of intending to GUARANTEE the truth of its conclusion IF the premises are true. This makes it a powerful tool: If you can offer arguments where the conclusion MUST be true (if the premises are true) then you've given a knock-down argument.

To see how it works, we must first separate the *form of the argument* from the *content*. Let’s take this example:

\[ P_1. \text{If stealing harms people, then it is morally wrong.} \]
\[ P_2. \text{Stealing does harm people.} \]
\[ C. \text{Therefore, stealing is morally wrong.} \]

The first premise is a conditional statement (an “if-then” statement). Break the proposition (the statement that can be true or false) of the antecedent (“stealing harms people”) from the proposition of the consequent (“it is morally wrong”) and translate them into variables. Vaughn uses \( p \) and \( q \) for the variables, I often use \( A \) and \( B \), but they are variables so you can use any symbol or letter as long as you specify what they refer to. Here is the FORM of the argument:

\[ P_1. \text{If } p \text{ then } q \]
\[ P_2. p \]
\[ C. q \]

Where \( p \) is “stealing harms people” and \( q \) is “stealing is wrong.”
VALID ARGUMENTS.

When we are talking about VALIDITY, then we are looking at the STRUCTURE or FORM of the argument, not the content. Forget for the moment about the content of the argument (forget that the above argument is stealing and moral statements) and just look at the structure of the argument. The argument above has the form of *modus ponens*, which is a valid argument. What it means to say an argument is valid is that *IF the premises are true, then the conclusion is GUARANTEED to be true*. To test if an argument is valid, you must first ASSUME that the premises are true. That is, if it is true that “if stealing does harm people then it really is morally wrong”, then you know that if someone steals, then they are doing something that is morally wrong. Assume that the premises are true, and you’ll see that the conclusion is guaranteed to be true. I must stress that to test validity you MUST ASSUME THE PREMISES ARE TRUE in order to see if the conclusion follows.

As another example, suppose I say that I went outside and saw that the road was wet and I claimed that it rained. We can form an argument as follows:

P1. If the road is wet, then it rained.

P2. The road is wet (by empirical evidence, since I saw it was wet).

C. Therefore, it must have rained.

Now, you might be thinking that this argument is false, since the road could be wet because someone sprinkled water on it instead of it raining. But forget about whether the statements are true about the world for the moment. ASSUME THE PREMISES ARE TRUE. That is, assume that “if the road is wet, then it rained” and “the road is wet.” IF it is absolutely true that “if the road is wet then it rained,” then it can’t fail to be the case that it rained given that the road is wet. The argument is valid.

There are two forms of valid arguments:

**Modus ponens:**

P1. If A then B.

P2. A.

C. Therefore, B.

**Modus tollens:**

P1. If A then B.

P2. Not B.

C. Therefore, not A.

Let’s try a *modus tollens* argument. Suppose I say to you that “If the road is wet, then it rained.” Now suppose the weather man tells me that it did not rain. Then I know for certain that the road is not wet. The *modus tollens* is follows:

P1. If the road is wet, then it rained. (If A then B)

P2. It did NOT rain (Not B).

C. Therefore, the road is NOT wet. (Not C).

Remember to assume that the premise is true, that if the road is wet then it is because it rained. If you are thinking that the road still could be wet because someone sprinkled water on it, then it changes the first premise,
that is, it would NOT be true that “if the road is wet, then it rained.” Your counterexample showed rather, that “if the road is wet then it rained OR that someone sprinkled water on it.” But you cannot change the premises. Assume that the premises are true, and you will see that the conclusion is guaranteed to be true.

Let’s try another one:

P1. All men are mortal.

P2. Socrates is a man.

C. Therefore, Socrates is mortal.

The form is:

P1. All A’s are B’s

P2. A.

C. Therefore, B.

This argument has a modus ponens form which is a valid deductive argument.

Here is another argument:

P1. All men are mortal.

P2. Socrates is NOT mortal.

C. Socrates is NOT a man.

This is a modus tollens argument, which is a valid deductive argument. Again, if you assume that it is true that “all men are mortal” and “Socrates is not a man” then you know for sure that Socrates cannot be a man.

A valid argument cannot have true premises and a false conclusion. There are no exceptions. One thing you want to do is to form possible counterexamples, which are examples that try to show that the premises can be true but the conclusion is false (or the conclusion doesn’t follow). Counterexamples will be abundant when you are faced with INVALID kinds of arguments, but they will be impossible to do with valid forms. To form counterexamples you must: 1. Keep the premises true (that is, it is true that “all men are mortal” and “Socrates is NOT mortal” meaning that he is immortal or can live forever). But 2. Falsify the conclusion. That is, say that “Socrates IS a man.” Now you have a counterexample where Socrates is NOT mortal but he IS a man. This counterexample DOES NOT work, however, because it is a direct contradiction to the first premise, which we must assume is true. That is, the first premise says that “All men are mortal,” so it can’t be the case that we have a man who is NOT mortal. Since our counterexample fails, we have further support that our original argument is valid.

Here is another argument. Is it a modus ponens or a modus tollens?:

P1. If my fridge is running, then the contents are cold.

P2. My fridge is running.

C. Therefore, the contents are cold.
Translate the propositions into variables as:

P1. If A then B.

P2. A

C. Therefore, B.

Where A is “my fridge is running” and B is “the contents are cold.” It is a modus ponens, valid, deductive argument. Try to form a counterexample, by keeping the premises true but the conclusion false. (I.e. if you say that “the potato salad is warm, yet the fridge is still running” then you've directly contradicted the first premise.) You will not be able to do so—that is the power of valid deductive arguments.

Another argument:

P1. If the cat is on the mat, then she is asleep. (If A then B)

P2. The cat is NOT asleep. (Not B)

C. Therefore, the cat is not on the mat. (Not A)

It is a modus tollens, valid, deductive argument.

INVALID ARGUMENTS.

Let’s talk about invalid forms now. An invalid argument uses bad forms of reasoning, since the conclusion does NOT follow from the premises. There are two INVALID deductive argument forms:

**Denying the antecedent:**

P1. If A, then B.

P2. Not A.

C. Therefore Not B.

**Affirming the consequent:**

P1. If A, then B.

P2. B

C. Therefore, A.

Here is an example of “Denying the antecedent”:

P1. All Cat’s are mammals ( If A then B, where A= cats and B= mammals)

P2. This is not a cat (it’s a dog). (Not A)

C. Therefore, this dog is not a mammal. (Not B)

You can see from the content that the conclusion is clearly not true, even if the premises might be. You can reason through this argument by understanding that the first premise says something about cats. That is, it gives a property or a condition about cats, but it gives no rules about dogs. Therefore, when you say anything about something other than a cat, then you cannot apply the condition in the first premise to it. You cannot conclude
from the first premise that a dog, frog, or anything other than a cat is NOT a mammal. The argument is “Denying the Antecedent,” invalid, and deductive. Notice that the reason the above argument form is called “Denying the Antecedent” is because the second premise claims that the antecedent of the conditional in the first premise is not true. The conditional statement, again, is “If it is a cat, then it is a mammal,” and the antecedent is “it is a cat” and the consequent is “it is a mammal.”

Here is another example of “Denying the Antecedent.”

P1. If my car is running, then there’s gas in the tank. (If A then B where A= “my car is running,” and B= “there’s gas in the tank”).

P2. My car is not running. (Not A)

C. Therefore, there’s no gas in the tank. (Not B)

It should be easy to come up with counterexamples. Remember to keep the premises true, but find examples where the conclusion does not follow. Here’s an example: It could very well be true that IF my car is running, then it does have gas in the tank (premise 1 is true). But my car could be broken for other reasons, it could be off, etc. and that is the reason why my car is not running (premise 2 is true). My car could be broken or off, but still have gas in the tank (Conclusion is false). So even though my car is not running, it does not mean that there is no gas in the tank. This counterexample keeps the premises true, but it shows that the conclusion is not guaranteed to follow. It is invalid.

Affirming the consequent:

P1: If the road is wet then it is raining. (If A then B where A = “the road is wet,” and B= “it is raining.”)

P2. It is raining. (A)

C. Therefore, the road is wet. (B)

It is “Affirming the Consequent,” Invalid, and Deductive. As you can probably figure out, the argument form of “Affirming the Consequent” will give a premise where the consequent of the conditional is true, and then it will try to conclude that the antecedent is true as well. This is bad reasoning.

It is easy to come up with counter examples: It could very well be true that if the road is wet then it is raining. It could also be true that it is raining. But suppose the road is under an overpass where it remains dry (not B, showing that the conclusion does not follow). This might be a tricky one, so think about it until you can see why it is a bad argument. Just keep in mind that the first conditional makes a rule about when the road is wet. It says nothing about other circumstances where it could be raining or where the road is dry.

Let’s test your knowledge: Valid or Invalid?:

P1. All bathrooms have toilets in them.

P2. This room has a toilet in it.

C: This room is a bathroom.
Lecture 2.1 DEDUCTIVE and INDUCTIVE LOGIC

Here is the form:

\[ P_1: \text{All A's are B's (or If it is an A then it is a B)} \]
\[ P_2: B \]
\[ C: A \]

It is Affirming the Consequent, Invalid, and Deductive. Here's a funny counterexample: Home Depot has a toilet in it, but it doesn't mean that it's a bathroom.

Here's a chart that you must memorize in order to know which forms are valid or invalid:

**Two valid deductive structures:**

<table>
<thead>
<tr>
<th>Modus ponens:</th>
<th>Modus tollens:</th>
</tr>
</thead>
<tbody>
<tr>
<td>If A, then B,</td>
<td>If A, then B,</td>
</tr>
<tr>
<td>A,</td>
<td>Not B,</td>
</tr>
<tr>
<td>Therefore B.</td>
<td>Therefore, not A.</td>
</tr>
</tbody>
</table>

**Two Invalid structures:**

<table>
<thead>
<tr>
<th>Affirming the consequent:</th>
<th>Denying the antecedent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If A, then B,</td>
<td>If A, then B,</td>
</tr>
<tr>
<td>B,</td>
<td>Not A,</td>
</tr>
<tr>
<td>Therefore A.</td>
<td>Therefore Not B.</td>
</tr>
</tbody>
</table>

**SOUNDNESS**

So far, we've been ignoring the content of the arguments, and focusing on the forms. But most arguments will be “bad” because they simply are not true. Soundess is about the truth of the content of the premises and conclusion. Sound arguments have premises that are true about the world. Importantly, though, since an argument is not good if it has an invalid structure, then a sound argument cannot be invalid. A sound argument is both valid and has true premises.

Let me repeat: You cannot have an invalid sound argument. An argument hast to be either *modus ponens* or *modus tollens*, AND all the propositions must be true about the world. Only when all these conditions are met, do you have a good sound argument.
Validity is not the same as soundness! Validity has to do with the structure and relations between the premises and conclusions that are made in an argument. Soundness has to do with whether the contents of the propositions are true.

An argument is valid if the conclusion necessarily follows from the premises. But you can have valid arguments that are not sound. For example:

- **P1.** All pigs are pink. (If it is a pig, then it is pink).
- **P2.** This is a pig.
- **C.** Therefore, this pig is pink.

This is a valid, modus ponens argument, but it is unsound— the propositions are not true about the world. There are brown pigs and grey pigs. Not all pigs are pink, so the first premise is false. Hence, it is a valid but unsound argument.

Here's another example of a valid, modus ponens, but unsound argument.

- **P1.** If the sun rises in the east, then pigs fly.
- **P2.** The sun rises in the east.
- **C.** Therefore pigs fly.

It is unsound because pigs don’t fly when the sun rises in the east.

There are also (*cringe*) invalid AND unsound arguments:

- **P1.** If Jones stands in the heavy rain without an umbrella, then Jones will get wet.
- **P2.** Jones is wet.
- **C.** Jones was standing in the heavy rain without an umbrella.

This one is “affirming the consequent” which is invalid, so it cannot be sound, even if it looks like the first premise is true. The first premise is not true, however as shown by a counterexample: Someone hosed Jones with water but he was not standing in the rain.

**HOW TO CHECK FOR VALIDITY AND SOUNDNESS.**

How to check for VALIDITY: Assume the premises are true. If the conclusion follows, it is a valid argument. You will know if the conclusion follows if it takes a *modus ponens* or *modus tollens* form. If the conclusion doesn’t follow, it is invalid.

How to check for SOUNDNESS: Is the argument valid? If so, then are all the premises true about the world? If the answer is yes to both of these, they you have a sound argument.

Form counterexamples to check for validity. A counterexample will show that an argument is invalid. If a counterexample shows that the premises can be true but the conclusion is false, then it is an INVALID argument.
Lecture 2.1 DEDUCTIVE and INDUCTIVE LOGIC

If your counterexample falsifies one of the premises, you’ve shown that the argument is UNSOUND. For example:

Premise: If the cat is on the mat, she is asleep.

Form a counterexample, that is true about the world. You might notice that the cat is on the mat, but she is chasing her tail and is not asleep. The counterexample shows that the premise is unsound. Make sure you use true propositions to form counterexamples.

INDUCTIVE ARGUMENTS:

Deduction involves inferring from a general law, to specifics. For example, if it is true that “If it is snowing, it is cold,” then you can infer that on ANY day that it is snowing, it will be cold TODAY. Or, from the statement, “All cats are mammals,” you can infer that my cat and your cat are both mammals. By contrast, Inductive arguments make inferences from more specific instances to more general types or laws. They do NOT aim to prove the conclusion is true. Rather they aim to show that the conclusion is mostly or probably true. They aim for reliability, not validity.

For example, you (or someone, scientists, people) might have observed the following, about specific swans they saw:

--This swan is white. . .
--this swan is white. . .
--this swan is white.

Now, they might conclude that “All swans are white.” We write the argument as follows:

P1. Swan 1, 2, 3, . . ., are white.
C. So ALL swans are white.

You can also appeal to the fact that all the swans that we have evidence about are white. From that evidence we infer that all swans are probably white. So you could say:

P1. All swans that were observed are white. (Empirical evidence).
C. All swans (even those not observed on Earth) are white.

Notice that the premise makes a claim about specific instances of swans that are observed. Then it concludes that ALL swans are probably white.

What makes it an Inductive argument is that it moves from specific instances to more general conclusions. Most arguments that deal with probabilities and statistical evidence are inductive. I.e. Empirical evidence shows that a billion known swans are white. Therefore, it is most likely true that “All swans are white.” Inductive arguments only aim to show that the conclusion is mostly or probably true. They aim to be STRONG or RELIABLE, which means that the stronger the evidence (or the more instances you have), the more likely it is that the conclusion is true.
For example, a survey team interviewed most students in a college and concluded that “75% of college students surveyed graduate.” This inductive argument is fairly reliable, since most students were surveyed. A small sample size is not reliable. For example, if you watch a commercial that says “Dentists recommend Crest,” and you find out that only two dentists were surveyed, then it is not a strong argument. Inductive arguments can be strong or weak, depending on the sample size.

You can generally spot an inductive argument if the conclusion contains the words: “Most,” “Sometimes” “Probably” “Some,” etc.

An example: “60% of the thousand people we surveyed in Utah said they voted for Obama. So, maybe 60% of the people in the United States voted for Obama.” The argument inferred from a sample of Utahans to a population in the U.S.

Most arguments we make and hear daily are inductive arguments. Induction involves inferring from several specifics, to a general conclusion. We infer that the sun will rise tomorrow because it has risen a billion times in the past. We infer that when we will eat an apple for lunch that it will satiate our hunger, because the last time we ate an apple we were satiated. We also do not aim to PROVE that things must be true. Most often, we rely on arguments that will probably or mostly be true.

**DEDUCTIVE OR INDUCTIVE?**

Vaughn explains how logical argument and persuasion are not the same thing. He says, "Presenting an argument is a way to demonstrate that a conclusion (a statement or belief) is warranted, that the conclusion is worthy of acceptance. This demonstration may or may not persuade someone to accept the conclusion; the argument's persuasiveness or lack thereof is a completely separate matter." (Vaughn, p. 22). The opposite case is true, of course, that someone may be persuaded by invalid arguments.

So, we have several measures of what can be considered "good" argumentation (I don't think there is any right answer to what counts as "good"). We have persuasiveness, but I don't think that's the best measure since people might be persuaded by what we intuitively think are "bad" arguments. Just think about how people might be persuaded by stories in the National Enquirer, where we think those arguments are bad.

Then we have "validity" which means that the conclusion of an argument is GUARANTEED to be true if the premises are (and it's in a valid deductive format). Valid arguments are pretty strong. If we assume the premises are true, then the conclusion must be true. However, the premises might not actually be true. Take for example:

P1. If McCain won the presidential election, we would have a republican president today.

P2. McCain won the presidential election.

C. Therefore we have a republican president.

This is a valid, modus ponens argument, but the premises are not true. So, someone might say it is a "good" argument, if they are only considering validity.

The better arguments are sound arguments, where it is valid and the premises are true. The above argument is not sound, of course, because premise 2 is blatantly false.
Here's a question: We learned about deductive and inductive arguments, which are both very useful. Do you think one kind of argument is better?

You might say that Deductive arguments might be better because they intend to guarantee the truth of the premises (whereas inductive arguments only intend to provide evidence that the conclusion is probably or mostly true). Scientists use deductive arguments. For example,

P1. The boiling point of water is 100 Celsius.
C. This pot of water will boil at 100 Celsius.
The reason why the argument is true is that it relies on laws (in this case a general law of heat) to show that any specific instance (a pot of water) will necessarily obey that law.

But then again, we would not know any laws (of motion, gravity, heat, etc.) without using inductive arguments. We know that the boiling point of water is 100 degrees Celsius because we conducted experiments a million times.

Here is the inductive argument:
P1. All the pots of water that were experimented with boiled at 100 degrees Celsius. (that is, water sample 1 boiled at 100 Celsius, water sample 2 boiled at 100 Celsius, water sample 3 ...)
C. All water will boil at 100 degrees Celsius.

Both inductive and deductive arguments are clearly useful.

Here is an important take-home note: **Validity and soundness ONLY apply to DEDUCTIVE arguments. Never say that an inductive argument is invalid or unsound.** The reason why is that **deductive** arguments intend to **prove** that the conclusion is true. Validity, recall, is where the conclusion is **guaranteed** to be true if the premises are. And, again, soundness only applies to valid arguments. But **inductive** arguments do not intend to guarantee that their conclusion is absolutely true if the premises are true. Rather, inductive arguments only intend to show that the conclusion is **probably** true. Since the conclusion is only **probably** true to a given degree, rather than absolutely true, then validity does not apply.

The following mindmap might help you organize the kinds of arguments, and where soundess and validity apply:
Questions on the Logic Exam.

For your Logic Exam (in week 3), you will be asked about 6 questions, where I supply the argument, and you must select all the options that apply. Here is an example of how the test questions will be formatted:

Multiple Choice (Select all that apply):

Deductive argument.
Inductive argument.
Valid.

*Modus ponens.*

*Modus tollens.*

Invalid.

“Affirming the Consequent.”

“Denying the Antecedent.”

Practice question 1.

P1. If my fridge is running properly, the contents are cold.

P2. But my potato salad is warm.

C. Therefore, my fridge must be on the fritz.

You will have to translate the argument yourself (on scratch paper) into:

P1. If A then B.

P2. Not B.

C. Therefore, not A.

Memorize the forms of *modus ponens*, *modus tollens*, “denying the antecedent,” and “affirming the consequent.” It is easiest to memorize the forms, although if you’ve forgotten them, you can always work out whether an argument is valid or not, by trying to apply counterexamples. Once you’ve translated the argument, you can see that it is:

Deductive, *modus tollens*, valid.
I do not ask whether the argument is sound or not, because there are many ways to argue whether a premise is true or not. In fact, philosophy is mostly about showing how certain arguments fail because they are unsound. As we shall see, philosophers come up with crazy counterexamples and thought experiments to show that an argument is unsound (or invalid, for that matter).

Assessment:

The following exercises will not be graded, but they will greatly help you build your logic skills.

* Be sure to download the “Logic Practice Quiz” that gives questions and answers in the format that will be asked in the exam. I give with detailed explanations for how to figure out the answers to each argument. I encourage you to work through an example thoroughly and post your reasoning on the Discussion Board.

* Come up with your own deductive and inductive arguments. Notice when people around you are making deductive or inductive arguments. Outline them and post them on the Discussion Board.


This is a very famous task. Be sure to read the “Further analysis” after they display the correct answers. It offers a test of your logic skills, and then points out that nearly 80% of people fail it because they reason according to “affirming the consequent.” See if you are using faulty logic as well.

* As explained in the “Further Analysis” page of the Wason Selection Test, the two Evolutionary Psychologists, Cosmides and Tooby, argue that humans do not have good reasoning skills about conditional rules. The exception, however, is when those rules applies to detecting when people are cheating in social situations (as the third question of the test demonstrates about detecting when underage people are drinking). They use the results of the test to show that we have evolved a “cheater-detection” module. When we are faced with arguments whose contents are non-social, then we irrationally reason by “affirming the consequent.” However, when the arguments (with the same structure) involve social situations where someone is trying to cheat someone else, then our “cheater-detection” reasoning mechanism kicks in and we are able to solve these arguments. They argue that we are born with a Cheater-detection reasoning skill, which was adapted in order to evolve as the social creatures we are. I encourage you to Google “Tooby and Cosmides” or “cheater-detection” and discuss it on the Discussion Board. What do you think of their theory? Why else do you think people use “affirming the consequent” so often, as shown by the “Wason Selection Task?”

* Be sure to ask questions about the various forms of deductive arguments. This may be difficult material, so don’t be afraid to ask clarification questions on the Discussion Board.